The Case for Case C Mass Transfer in the Galactic Evolution of Black Hole Binaries

Gerald E. Brown [a,1] Chang-Hwan Lee [b,2]

[a]*Department of Physics and Astronomy,*
State University of New York, Stony Brook, NY 11794-3800

[b] *Department of Physics and*
Nuclear Physics & Radiation Technology Institute (NuRI),
Pusan National University, Busan 609-735, Korea

Abstract

Earlier works, which we review, have shown that if the Fe core in a presupernova star is to be sufficiently massive to collapse into a black hole, earlier in the evolution of the star the He core must be covered (clothed) by a hydrogen envelope during He core burning and removed only following this, in, e.g. common envelope evolution. This is classified as Case C mass transfer. These previous arguments were based chiefly on stellar evolution, especially depending on the way in which ^{12}C burned.

In this work we argue for Case C mass transfer on the basis of binary evolution. The giant progenitor of the black hole will have a large radius $\sim 1000 R_\odot$ at the end of its supergiant stage. Its lifetime at that point will be short, ~ 1000 yrs, so it will not expand much further. Thus, the initial giant radius for Case C mass transfer will be constrained to a narrow band about $\sim 1000 R_\odot$. This has the consequence that the final separation a_f following common envelope evolution will depend nearly linearly on the mass of the companion m_d which becomes the donor after the He core of the giant has collapsed into the black hole. The separation at which this collapse takes place is essentially a_f, because of the rapid evolution of the giant. (In at least two binaries the black hole donor separation has been substantially increased because of mass loss in the black hole formation. These can be reconstructed from the amount of mass deposited on the donor in this mass loss.)

We show that the reconstructed preexplosion separations of the black hole binaries fit well the linear relationship.

1 Introduction

The black hole Soft X-ray Transient (SXT) A0620−00 consisting of an $\sim$ $11 M_\odot$ black hole and $\sim 0.7 M_\odot$ K-star companion was evolved theoretically by De Kool et al. (1987) with Case C mass transfer; i.e., the common envelope evolution in which the companion removed the hydrogen envelope of the giant black hole progenitor through spiral-in took place after He core burning was completed in the giant. In Table 1 of Lee, Brown, & Wijers (2002) (denoted as LBW), the seven SXTs with shortest periods had K- or M-star companions and the unclassified companion in XTE 1859+226 may also well be K or M because of its short period $P = 0.380$ days. The progenitor binaries of these would have involved $\sim 25 M_\odot$ giants and $\sim 1 - 2 M_\odot$ companions, the latter having had some mass stripped off by the black hole. In other words, all of the shortest period SXTs are successfully evolved with the *same* Case C mass transfer.

We emphasized in Lee & Brown (2002) that in Case C mass transfer the orbital separations for the above progenitor binaries in Roche Lobe contact are at $\sim 1700 R_\odot (\pm \sim 10\%)$, ~ 8 AU, for ZAMS $20 M_\odot$ black hole progenitor and $1 M_\odot$ companion star. Progenitors with more massive companions and the larger initial separation necessary for Case C mass transfer could have removed the H-envelope of the giant with spiral-in to larger final separations a_f, since their drop in gravitational energy of the more massive companion is then sufficient to remove the envelope, and we shall see that this is indeed what happens.

In LBW we listed Nova Scorpii and IL Lupi as undergoing mass transfer while in main sequence. Beer & Podsiadlowski (2002) have carried out a detailed, convincing numerical evolution of Nova Scorpii, showing that the orbit has widened substantially under nearly conservative mass transfer. Podsiadlowski et al. (2002) (denoted as PRH) have recently extended such calculations to the other binaries with evolved companions, showing that they all began mass transfer in main sequence, although V404 Cyg, J1550−564 and probably GRS 1915+105 will have progressed beyond main sequence. (As noted later, we differ with PRH in our suggested evolution of Cyg X-1.) The PRH calculations generally support the schematic LBW calculations of mass transfer, but have the added advantage that by beginning the transfer in main sequence, sufficient mass can be transferred in the traditional sub Eddington limit. Whereas we do not believe this to be necessary in the case of black holes, seeing no reason why the accretion across the event horizon could not be substantially hyper Eddington (and PRH also covers this case, as a possibility) the standard PRH

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[1] Ellen.Popenoe@sunysb.edu
[2] clee@pusan.ac.kr

scenario allays the fear of the greatly hyper Eddington scenario which may go against "accepted wisdom".

In Case C mass transfer there is a great regularity expressed in the roughly linear dependence of companion mass on orbital separation of the giant black hole progenitor and companion on the companion mass

$$a_f \propto \frac{M_d}{M_\odot} \left(\frac{M_{\mathrm{giant}}}{M_\odot}\right)^{-0.55} R \tag{1}$$

following the spiral-in stage which removes the envelope of the giant (Lee, Brown & Wijers 2002). We derive Eq. (1) in the Appendix. Except for the roughly square root dependence on giant mass, this relation is linear. Here the companion (donor) mass is labelled M_d, a_f is the separation of the He-star, companion binary following spiral-in in common envelope evolution, and R is the initial radius of the giant at the start of common envelope evolution. The dependence on M_{giant} is weak, the interval

$$20M_\odot < M_{\mathrm{giant}} < 30M_\odot \tag{2}$$

being used by Lee, Brown & Wijers (2002). The term depending on giant mass originates from the term $M_{\mathrm{He}}/M_{\mathrm{giant}}^2$ in common envelope evolution. The relation Eq. (1) is particularly useful because, as we shall argue, R is nearly constant, $\sim 1000R_\odot$, to within $\sim 10\%$. Because the giant evolutionary time is so short, a_f is essentially the preexplosion separation of black hole and donor.

We thus have three classes:

(i) The 8 AML (angular momentum loss) SXTs with K or M-star main sequence companions come from binaries which overfill their Roche Lobe during spiral in, as discussed in LBW. Their periods are decreased as they transfer mass to the companion black hole, as they lose angular momentum by magnetic braking and gravitational waves.

(ii) The next six SXTs which established Roche contact while in main sequence, some of them having evolved beyond.

(iii) The special case of the continuously shining Cyg X-1 which we place just before its Roche Lobe, the companion now undergoing unstable mass transfer to its lower mass companion black hole.

Interestingly, we find that the division between the unevolved main sequence class (i) and evolved companion (ii) is given accurately by Fig. 2 of de Kool et al., who plot the mass of the companion which undergoes angular momentum loss by gravitational waves and magnetic braking, both as functions of time. They obtain the companion mass of $2M_\odot$ as giving the division. We find this to be true for $M_{\mathrm{giant}} = 20M_\odot$ in Eq. (1).

Note that the binaries with late main sequence companions Nova Scorpii and IL Lupi are special in that these binaries have experienced large mass loss, which can be explained as in LBW by magnetohydrodynamic effects, not included in the evolution discussed here. One may wonder why just there two binaries, with ZAMS companion masses, have lost a sizable fraction of their progenitor He star masses, whereas there is no sign of a kick outwards in separation from copious mass loss in the Class (i) SXTs.

The above regularity shows immediately that at least the first class with K and M companions must have a very large a_i (LBW find $a_i \sim 1700R_\odot$ for a $20M_\odot$ black hole progenitor with $1M_\odot$ companion) corresponding to a giant radius of $\sim 1000R_\odot$, so that the binding energy of the giant envelope, which decreases inversely with its radius, is small enough to be furnished by the drop in gravitational binding energy of the low-mass companion as it spirals in to its Roche Lobe.

As developed in many papers by Brown and collaborators, most lately in Lee & Brown (2002) and LBW, and backed by evolutionary calculations by Brown et al. (2001) and PRH, the above delineation into three classes, depending upon companion mass, can be understood if the giant is required to finish (or nearly finish) He core burning before common envelope evolution takes place; i.e., if the mass transfer is essentially Case C.

In Sec. 2, we discuss the stellar evolution necessary to produce models which allow Case C mass transfer for ZAMS $20-30M_\odot$ stars. In Sec. 3, we review the role of carbon burning. In Sec. 4, we discuss that Case B mass transfer would not only allow too much of the He envelope to blow away and leave too much ^{12}C after He core burns, but also is disfavored by the population of SXTs. In Sec. 5, we compare our approach with other works, especially those by PRH, and discuss population synthesis of Case B mass transfer. We summarize our conclusion in Sec. 6.

2   The Case for Case C Mass Transfer

In LBW we found that the Schaller et al. $20M_\odot$ star had the characteristics we desire for Case C mass transfer, but that the latter was not possible for their $25M_\odot$ star. We therefore constructed "by hand" models in which the stellar radius as function of burning stage had a similar shape to the $20M_\odot$ star, all the way up to $30M_\odot$.

We show in Fig. 1 the results of the Schaller et al. (1992) stellar evolution for a ZAMS $20M_\odot$ star. It is seen that the main increase in radius comes after the start of He core burning (which begins while H shell burning is still going
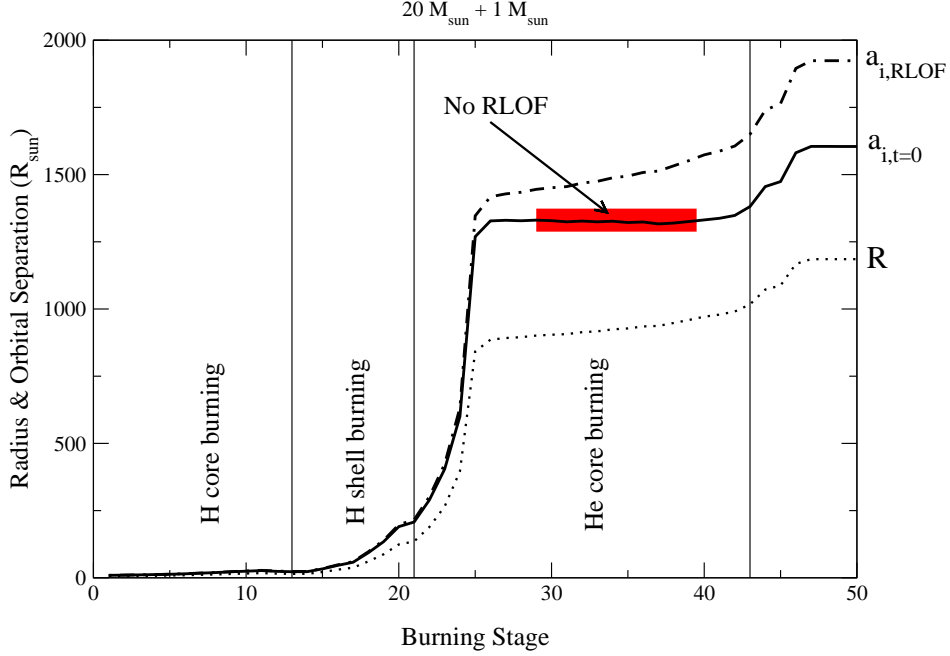
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Fig. 1. Radius of black hole progenitors (R) and the initial orbital separations (a_i) of the progenitors of X-ray transient binaries with a $1M_\odot$ companion. The burning stage in the x-axis corresponds to that of Schaller et al. (1992). **A)** The lower dotted curves (R) corresponds to the radius of the black hole progenitors taken from Schaller et al. (1992). That for the $25M_\odot$ star is similar but for the $30M_\odot$ the radius does not increase following the end of He core burning. **B)** From the mass of the primary at the tabulated point one can calculate the semimajor axis of a binary with a $1M_\odot$ secondary in which the primary fills its Roche Lobe, and this semimajor axis is shown in the upper dot-dashed curve ($a_{i,\mathrm{RLOF}}$). **C)** The solid curves ($a_{i,t=0}$) correspond to the required initial separations after corrections of the orbit widening due to the wind mass loss, $a_{i,t=0} = a_{i,\mathrm{RLOF}} \times (M_p + M_d)/(M_{p,0} + M_d)$ where M_p is the mass of the black hole progenitor at a given stage and $M_{p,0} = 20M_\odot$ is the ZAMS mass of the black hole progenitor. Primaries at the evolutionary stages marked by the shaded area cannot fill their Roche Lobe for the first time at that stage, but have reached their Roche Lobe at an earlier point in their evolution.

on). With further He core burning there is a flattening off of the radius versus burning stage and then a further increase in radius towards the end of and following He core burning. Our model requires that mass transfer take place during this last period of increase in radius, so that the orbital separation ($\sim 3/2$ of the giant radius) is well localized $a_{i,\mathrm{RLOF}} \sim 1700R_\odot$ at the time of Roche Lobe contact, or $a_{i,t=0} \sim 1500R_\odot$ initially, the difference due to mass loss by wind, with accompanying widening of the orbit.

It is made clear in LBW and Lee & Brown (2002) that for Case C (or very

late Case B) the radii of the relevant stars must have the following behavior, as shown in Fig. 1.

(i) They must increase rapidly in radius with hydrogen shell burning and with the early He core burning, which begins while the hydrogen shell burning is still going on.

(ii) The radii must flatten off, or actually decrease with further He core burning. This is so that if the companion reaches the Roche Lobe it will reach it before or early in He core burning. Then according to Brown et al. (2001a) the He core made naked by common envelope will mostly blow away by the strong Wolf-Rayet type winds, and the final Fe core will be too low in mass to collapse into a high mass black hole.

(iii) The third (obvious) characteristic is that the stellar radius must grow following He core burning, because the massive star must be able to reach its Roche Lobe during this time. The massive star has only $\lesssim 10^4$ years of its life left, so wind losses no longer can carry much of it away.

Portegies Zwart et al. (1997) have pointed out that wind loss from the giant preceding common envelope evolution is important and we follow their development in identifying the "No RLOF" part of the curve in Fig. 1. Because of the wind loss the binary widens. The Roche Lobe overflow will take place during the very rapid increase in radius of the giant in the beginning of He core burning, or in very late Case B or in Case C mass transfer. The binary has widened too much by the time the giant has reached the flat part of the R vs stage curve. In fact, it cannot transfer mass during this stage, because it will have already come to Roche contact during early Case B. This is made clear by the shaded area in the solid line in Fig. 1. Brown et al. (2001c) have shown that the SXTs with main sequence companions can be evolved with a $1 - 1.25 M_\odot$ companion mass, so the above results may be directly applicable. For higher mass companions, this shaded area becomes smaller because the effect of the winds is smaller. This makes the intermediate Case B mass transfer possible. However, in this case, high-mass black holes may not form because the Fe core is not massive enough to form high-mass compact objects as we discussed above (Brown et al. 2001a). Furthermore, as in Fig. 2, the probability of the intermediate Case B mass transfer is small compared to that of Case C mass transfer.

Now, in fact, the curve of radius vs burning stage for the next massive star, of ZAMS $25 M_\odot$, by Schaller et al. (1992) does not permit Roche Lobe contact during Case C at all, the winds having widened the binary too much by the time the giant radius begins its last increase in late He core burning. In the ZAMS $30 M_\odot$ star of Schaller et al. (2002), there is no increase in R at this stage, so Case C mass transfer is not possible.

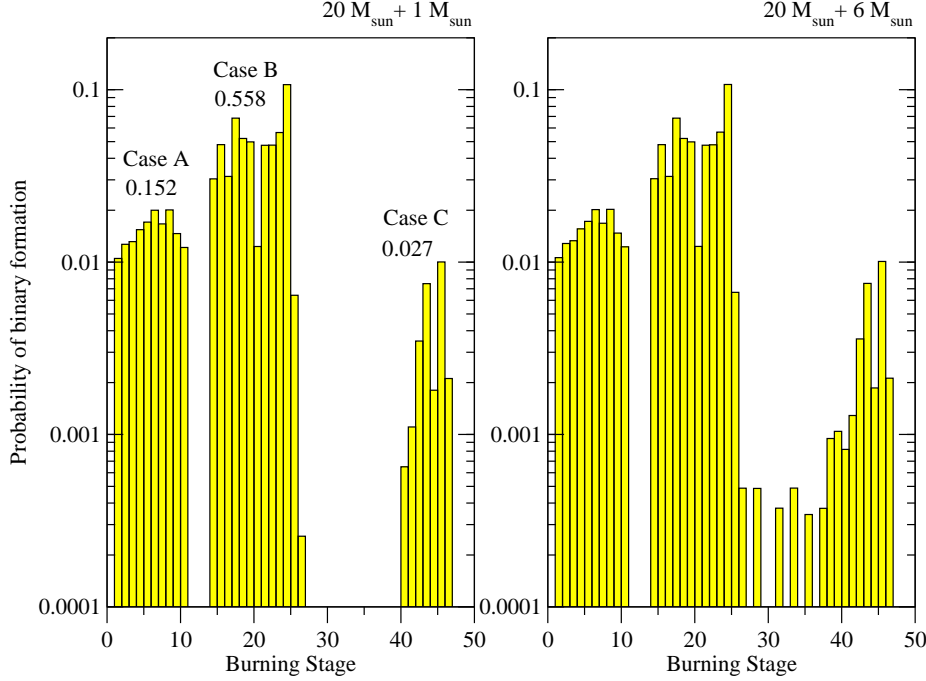The lack of increase in R for the more massive stars is due to the cooling

Fig. 2. Probability of initial binary formation, in which the Roche Lobe overflow starts between the two adjacent burning stages of the $20M_\odot$ ZAMS star. The burning stages are the same as in Fig. 1. The probability (logarithmic distribution of initial binary separation) is given by $P = \log(a_{n+1}/a_n)/7$ where the initial binary separation $a_{i,t=0}$ is between a_n and a_{n+1}, and the logarithmic distribution is normalized by the total logarithmic interval "7" of Bethe & Brown (1998). Three different cases of mass transfer are marked by Case A, B, and C. The numbers for each case in the left panel are the total of the probabilities in each case. In the left hand panel, the radii for the Case B mass transfer between stage 14 and stage 27 are $R = 22-892R_\odot$ with the corresponding initial binary separation $a_{i,t=0} = 33 - 1330R_\odot$. For Case C mass transfer between stages 40-47, $R = 971-1185R_\odot$ and $a_{i,t=0} = 1331-1605R_\odot$. With a $6M_\odot$ companion, the intermediate Case B mass transfer is possible as in the right panel. However, the total probability for the intermediate Case B mass transfer is $\sim 10\%$ of that for the late Case B and Case C mass transfer.

effect by strong wind losses. As shown by Lee et al. (2002) giant progenitors as massive as $30M_\odot$ are necessary as progenitors of some of the black holes in the SXTs, especially for the binaries with evolved companions, in order to furnish the high mass black hole masses. These authors reduce wind losses by hand, forcing the resulting curve of R vs burning stage to look like that for a ZAMS $20M_\odot$ shown in Fig. 1 during the He core burning where the effect of wind loss is important. In other words, in order to get the observed regularities in the evolution of SXTs, especially Eq. (1) which gives the linear dependence on M_d of the preexplosion separation of the binary, we must manufacture R

vs burning stage curves for which mass transfer can be possible both early in Case B and in Case C. With early Case B mass transfer, or intermediate Case B mass transfer if it occurs, the winds during He core burning are so strong that not enough of an Fe core is left to result in a high-mass black hole, rather, a low-mass compact object results (Brown et al. 2001a).

This story is somewhat complicated, but there have been many years of failures in trying to evolve black holes in binaries without taking into account the effects of binarity (mass transfer in our model) on the evolution. On the other hand, de Kool et al. (1987) had no difficulty in evolving A0620−00 in Case C mass transfer. The necessity in a similar evolution for the other black hole binaries was, however, not realized at that time.

3 Dependence on the $^{12}C(\alpha, \gamma)^{16}O$ Rate

Brown et al. (2001a) showed that the mass at which single stars went into high-mass black holes was determined by the $^{12}C(\alpha, \gamma)^{16}O$ rate. For the Woosley rate of 170 keV barns, stars from $8 - 18M_\odot$ would go into neutron stars, the narrow range from $18 - 20M_\odot$ into low-mass black holes (We believe 1987A to be an example.) and the stars from $20M_\odot$ on up to a maximum mass determined by wind losses, possibly $\sim 30M_\odot$ into high-mass black holes.

The main conclusion of Brown et al. (2001a) was that the massive star must be clothed by its H envelope during most, if not all, of its He core burning, if the core is to be massive enough so as to collapse into a high-mass black hole.

Schaller et al. used ~ 100 keV barns for the $^{12}C(\alpha, \gamma)^{16}O$ rate, and we can check that their central ^{12}C abundance following He core burning goes down to $\sim 15\%$ for their $25M_\odot$ star. In fact their $25M_\odot$ star does expand quite rapidly just at their stage 43, the end of He core burn. However, large wind losses cause the binary to widen too much for Case C mass transfer, and these must be cut down somewhat as done by LBW if Case C is to be made possible.

One consequence of the skipping of convective carbon burning is that the remaining lifetime of the core should be substantially foreshortened. Whereas convective carbon burning takes hundreds of years, neon and oxygen burning take only $\sim$ one year. The interpolation from ^{12}C to ^{16}O burning via radiative and shell ^{12}C burning and neon burning, which remains even when the central ^{12}C is less than 15%, will smooth out any abrupt change, but the foreshortening should none the less be appreciable. It lessens the time available for tidal interactions in the He-star, donor binary lifetime.

Our considerations apply to Galactic metallicity. With low metallicity, the

opacity is less and winds would not be expected to blow off naked He envelopes. Thus, Case A, AB or B mass transfer might not be expected to lead to only low-mass compact objects. The LMC with metallicity about 1/4 Galactic, has two continuously shinning X-ray binaries, LMC X-1 and LMC X-3, even though the total LMC mass is only $\sim 1/20$ of Galactic.

There is an important caveat to the large expected effect from lower metallicity and stronger winds. As discussed in Brown et al. (2001a) (see their Table 2) the mass loss rate has to be lowered by a factor of 3 from the preferred rate (which fits the fractional period change $\dot{P}/P$ in V444 Cyg) before the convective ^{12}C burning is skipped (with a central 12% ^{12}C abundance). In fact, even then (Fryer et al. 2002) the compact core is only 1.497, only large enough to collapse into a low-mass compact object. But, in the Fryer et al. (2002) calculations, the compact core is brought back up to $10.7 M_\odot$ by fallback, sufficient for collapse into a high-mass black hole (as in the $5.2 M_\odot$ remnant obtained when the mass loss rate is cut down by a factor of 2, rather than 3). However, (Brown et al. 2000; LBW) in a binary magnetohydrodynamic effects should help expel the outer matter in the explosion, cutting down the fallback.

From the above one can see that even cutting winds down will not necessarily make Case B mass transfer possible. Perhaps more important is the lack of ^{4}He needed to burn the last ^{12}C left. As the triple alpha reaction depends on the third power of the helium mass fraction it loses against the ^{12}C$(\alpha,\gamma)^{16}$O reaction toward the end of central helium burning; i.e., carbon is mostly burned rather than produced toward the end of central helium burning. That switch typically appears at a central helium mass fraction of $\sim 10 - 20\%$. Most importantly, as can be seen from the central carbon abundances at the end of He burning, which decreases from 35% to 22% with the lowering of wind losses by a factor of 6 (Fryer et al. 2002) the He fraction is too low to burn the final carbon. Only with the 6-fold reduction in wind from the Woosley, Langer, & Weaver (1995) rate (which is 3-fold from our preferred value)[3] is convective carbon burning skipped. (With a 4-fold lowering from WLW, the convective carbon burning goes on for 500 years.)

In the clothed stars, on the other hands, the growth of the He core and accompanied injection of helium after this time leads to a further decrease of carbon as compared to the bare helium cores that do not have this additional supply of helium. We believe the above may be the most important difference between naked and clothed He cores.

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[3] Half of the Woosley, Langer & Weaver (1995) rate.

4 Evolutionary Consequences If Case B Were Possible

We see from Eq. (1), or equivalently, Eq. (A.3) that the preexplosion separation a_f scales linearly with M_D, a relation that was used in LBW to evolve all binaries with evolved companions. Note that the range of a_i also depend on the donor masses through the changes in Roche Lobe radii as in Fig. 3. In LBW we used this scaling, which also followed from the Webbink common envelope evolution, and showed that the evolution of all of the SXTs could be understood in terms of it.

During H shell burning and He core burning the radius R of the giant increases rapidly up to $\sim 892R_\odot$. (During the increase from $\sim 892R_\odot$ to $971R_\odot$ the wind losses widen the orbit at such a rate that there is no RLOF as shown in Fig. 1. The decrease in the $a_{i,t=0}$ curve is even more pronounced in the curve of Heger (private communication, 2000). This may change with decreased wind losses, but we expect any increase in the $a_{i,t=0}$ to be small, and neglect it here. Consequently, Case B mass transfer could be early, taking place with H shell burning or early He core burning.

Suppose Case B mass transfer takes place during the stages between 15 and 27 as in Fig. 2. Certainly it can, although we say the results will be a binary with a low-mass compact object. It would most likely do so for radii from ~ 22 to $892R_\odot$, and the corresponding initial binary separation from $\sim 33R_\odot$ to $1330R_\odot$. We set the lower limit to be the radii at the stage 15 following the gap between Case A and Case B mass transfer in Fig. 2. For the total binary logarithmic interval we take the 7 of Bethe & Brown (1998). With the above $33 - 1330R_\odot$ the fractional logarithmic interval is $\ln(1330/33)/7 \sim 0.53$ whereas for Case C mass transfer it is $\ln(1604/1331)/7 = 0.026$. Thus, for a logarithmic distribution of binaries, Case B mass transfer is favored by a factor ~ 20. What would the consequences of this be, assuming it to be possible ?

First of all let us consider SXTs like V4641 Sgr which is just beginning to cross the Herzsprung gap; this consideration also includes GRS 1915+105 which was shown by LBW to be a late V4641 Sgr on the other side of the Herzsprung gap. Both had the large, ZAMS $\sim 6.5-8M_\odot$ companions. V4641 Sgr has at present radius $R = 21.3R_\odot$. and from the closeness of black hole and companion masses, could not have been narrower than $\sim 20.5R_\odot$ $(= 21.3 \times (\frac{9.61 \times 6.53}{8.07^2})^2 R_\odot)$ at which separation the companion and black hole mass would have been of nearly equal mass, which could have been as massive as $8M_\odot$ originally.

By way of example of how Case B mass transfer might function, we consider binaries with $M_{\mathrm{He}} = 11M_\odot$ and $M_{\mathrm{D}} = 8M_\odot$; i.e., binaries similar to our reconstructed V4641 Sgr at the time of black hole formation, as an example. The orbital separation for Roche Lobe overflow is $\sim 13.2R_\odot$, taking the donor
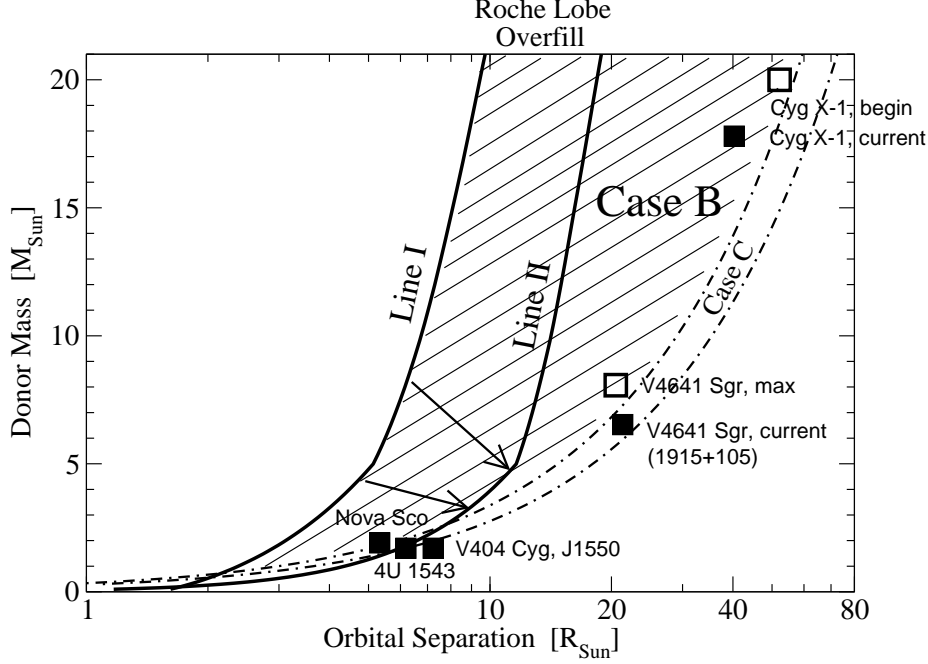
Fig. 3. Orbital separations after common envelope evolution for Case B and Case C mass transfer. Dot-dashed lines are the limits for the Case C mass transfer with $M_p = 30M_\odot$, $M_{He} = 11M_\odot$, and $\lambda\alpha_{ce} = 0.2$ (see LBW). All the area to the left of the left dot-dashed line is Case B mass transfer (shaded area). Line I is the sum of the radius of the companion and that of the He core which is assumed to be $1.5R_\odot$. Line II is the orbital separation corresponding to the Roche Lobe overfill right after the spiral-in during the common envelope evolution. The companion stars in binaries between Line I and II will be inside their Roche Lobe (Roche Lobe overfill) when they finish common envelope evolution, and they will be pushed out with mass transfer as indicated by arrows or they will lose in the common envelope evolution (see Fig. 4). Those binaries ($M_{donor} > 2.5M_\odot$) between Line II and the left boundary of Case C will be outside of the common envelope even with Case B mass transfer. Reconstructed preexplosion orbital separation and black holes masses of SXTs with evolved companions are marked by black squares (refer to Fig. 11 of LBW.) If the high mass black hole formation in Case B mass transfer were possible, the probability of observing them in Case B is ~ 7 times larger than in Case C. However, for the donor masses $\gtrsim 2M_\odot$, we see no SXTs in Case B, while we have two observations, V4641 Sgr and GRS 1915+105 in which the reconstructed data is consistent with Case C. We have put in both the reconstructed data with maximum initial black hole mass (open square), and the present position of V4641 Sgr (filled square) in order to show the uncertainty in reconstruction. The small change in orbital separation shows this binary to give an excellent fiducial preexplosion separation. Because of the long period, mass loss in the explosion will be low (LBW). Cyg X-1 may have had the preexplosion separation shown by the open box; its current separation is shown by the filled box. Although the a_f is linear with companion mass M_d, the curves delineating Case C mass transfer curve up when the orbital separation is plotted logarithmically.

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radius to be $4.5R_\odot$. The radius of an $11M_\odot$ He star is $1.5R_\odot$, so the sum of donor radius plus He-star radius is $6R_\odot$. If the binary separation is smaller than this, it will merge during the evolution. So the range of orbital separations for Roche Lobe overfill after common envelope evolution is $\sim 6 - 13R_\odot$. This means that if the companion star spirals in from anywhere between an initial 823 and $1770R_\odot$ it will overfill its Roche Lobe.[4] It will then transfer mass to the He star until it fits into its new Roche Lobe with reduced mass. [5] Because of the substantial logarithmic interval $\ln(1770/823)/7 = 0.11$, nearly 1/5 of the entire Case B logarithmic interval, or nearly 4 times the entire Case C logarithmic interval. Conditions of Roche Lobe overfill for various donor masses are summarized in Fig. 3.

Now consider the case that the binary ended up with the orbital separation of $6R_\odot$ after common envelope evolution. Since the separation for the Roche Lobe filling is $4.5R_\odot/0.35 \sim 12.9R_\odot$, the donor will lose mass until the radius of the donor fits its Roche Lobe. If we assume conservative mass transfer, the mass transfer will stop before the donor reaches $\sim 4M_\odot$ as in Fig. 4. Furthermore, if the explosion occurs before the donor radius fits its Roche Lobe, the orbit will be widened during the explosion. This will reduce the mass loss from the donor star, so the final reduced main sequence mass will be larger than $4M_\odot$ as indicated by arrow in Fig. 3.

The chance of seeing the SXTs with massive companions before they evolve is small, even if the high mass black hole formation is possible in Case B mass transfer, either because the binary will end up beyond its Roche Lobe on Line II or because the mass transfer during the early main sequence stage will be small if it ended up along the Line II and will increase the binary radius beyond the Roche Lobe. Note that the life time of main sequence with ZAMS mass $\gtrsim 2M_\odot$ is shorter than the time scale of the orbital separation (e.g., Fig. 2 of De Kool et al. 1987, without magnetic braking). Instead, they will become SXTs when they evolve. In the case of V4641 Sgr with initial companion mass $M_d = 8M_\odot$, $M_{\rm BH} = 8M_\odot$ we estimate that after common envelope evolution $a_f \sim 1.5R_L$. Since the radius more than doubles in late main sequence evolution (Schaller et al. 1992) it will reach its Roche Lobe before then. We thus find that all companions with masses $> 2M_\odot$, aside from that in Cyg X-1, establish Roche contact in main sequence. In Fig. 3, therefore, all binaries between Line I and the boundary of Case B and Case C will become SXTs with evolved companions, if high mass black hole formation in Case B mass transfer were possible. In that case, from Fig. 3, one can see that there should be ~ 8 times more SXTs with evolved companions (with

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[4] The initial orbital separation for Case C mass transfer is larger than those in Fig. 2 due to the larger radius of the companion star.
[5] The He star may accept some of the mass or it may be lost in common envelope evolution.
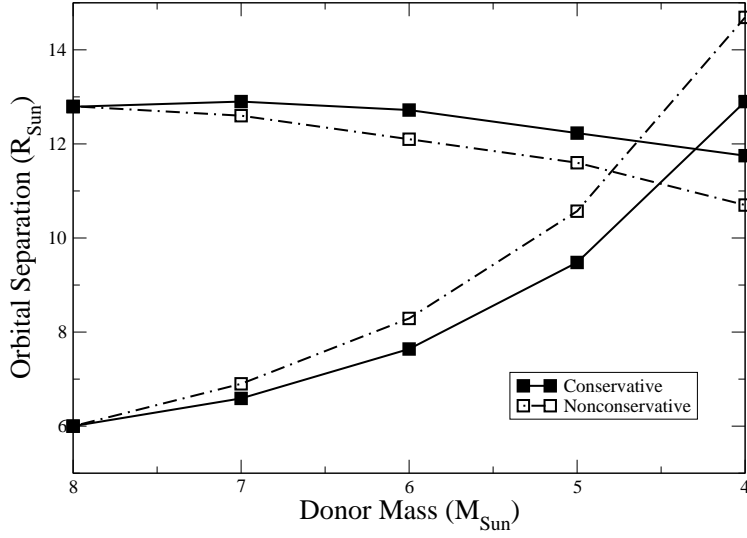
Fig. 4. Evolution of the orbital separations of the binary which overfill the Roche Lobe after common envelope evolution. We assumed the binary separation after common envelope evolution $a_f = 6R_\odot$ (left end point of lower curve), the sum of the companion and the He core radii (Line I in Fig. 3), with $M_{\text{He}} = 11M_\odot$ and $M_D = 8M_\odot$. Since the donor is inside its Roche Lobe, the outer envelope will be transferred to the He core until the donor fits its Roche Lobe. The lower curves correspond to the orbital separations during the mass transfer. The upper curves correspond to the outer radius for the Roche Lobe overfill (Line II in Fig. 3) with corresponding donor and He core masses. Roche Lobe overfill will occur for any a_f between $6R_\odot$ and the upper curve, and there will be additional lower curves corresponding to these different a_f. The mass transfer will continue until the lower curve reaches the upper one. Once the lower curve reaches the upper one, there is no further mass transfer and the orbital evolution will stop. In the conservative mass transfer, we assume that 100% of the mass lost from the donor is accreted onto the He star, and in the nonconservative mass transfer case, we assumed that all the transferred mass from the donor is lost. (It may be expelled in the common envelope evolution.) In both cases, the mass transfer will stop before the donor reaches $\sim 4M_\odot$ where the Roche Lobe is larger than the donor radius. In the same way, we will arrive $M_d \sim 3M_\odot$ ($1.5M_\odot$) if we start with $M_d = 4M_\odot$ ($2M_\odot$) as indicated in Fig. 3.

initial donor mass $> 2.5M_\odot$). On the other hand, from Fig. 3, we expect ~ 4 times more SXTs with companions in main sequence if Case B mass transfer were possible.

Chiefly we see from our discussion of possible Case B mass transfer in the SXT evolution that there would be no correlation between companion mass and preexplosion separation, since the possible initial separations a_i would be

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very widely spread. In many cases, the orbit following spiral-in would overfill its Roche Lobe and mass exchange or loss would spread out the companion masses, each binary filling its Roche Lobe. The validity of Eq. (1) depends on the possible post supergiant radii R being within a narrow range, consequently a narrow range in the preexplosion orbital separation a_i. We show in Fig. 3 that empirically the relation Eq. (1) is satisfied with our preferred common envelope efficiency $\lambda \alpha_{ce} = 0.2$ of LBW. Of course this depends on the reconstruction of preexplosion orbits by LBW, which generally is supported by PRH although they give a wide range of possibilities.

LBW noted that the evolution of Cyg X-1 also fits into our Case C mass transfer scenario as in Fig. 3. Assume the progenitor of the black hole to be a ZAMS $25 M_\odot$ giant (with $8.5 M_\odot$ He star which we assume to go into a black hole of the same mass because very little mass is lost in the case of such a long period[6]). Following the supergiant stage of the massive giant a ZAMS $20 M_\odot$ companion removes the envelope, coming to an a_f of $\sim 50 R_\odot$ ($= (20 M_\odot / 8 M_\odot) \times 20 R_\odot$), where $8 M_\odot$ and $20 R_\odot$ are the reconstructed black hole mass and a_f at the time of explosion in V4641 Sgr. We followed the linear scaling of a_f with M_d here. The companion now transfers $2.2 M_\odot$ to the black hole in unstable, but conservative mass transfer. This brings the separation a down to the present $40 R_\odot$.

5 Comparison with Other Works and Population Synthesis

As noted earlier, a comprehensive numerical evolutionary calculation has been carried out by Podsiadlowski et al. (2002; PRH), who also adopt case C mass transfer. Out approach as that in LBW is schematic, but there is substantial agreement between PRH and LBW on most aspects of mass transfer and the effects that follow from it. This is not surprising since LBW built their work on the earlier evolutionary calculation for Nova Scorpii of Beer & Podsiadlowski (2002).

PRH agree with LBW that present evolutionary calculations for giant ZAMS masses $> 20 M_\odot$ do not allow Case C mass transfer, and that these must be changed.

The LBW approch was to cut down on wind loss so as to make giants from ZAMS $20 - 30 M_\odot$ behave similarly to the Schaller et al. (1992) $20 M_\odot$ one.

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[6] A correction may have to be put in for mass loss in the explosion forming the black hole (Kaper et al. 1999) although the system velocity may be only $\sim 1/3$ the 50 km s^{-1} found there, depending on the O-star association (L. Kaper 2001, private communication).

It ascends the asymptotic giant branch with highly convective envelope near the end of evolution. PRH show in their Fig. 1 calculations performed without wind loss for a large range of ZAMS masses and they exhibit this behaviour.

We base our evolution on our Eq. (1) having established in LBW that it is consistent with present observational data, if the product $\lambda\alpha_{ce}$ in the Webbink (1984) common envelope evolution is set equal to 0.2. Possible deviations from our choice can be found from Fig. 3 of PRH for values of 0.5 and 0.08, 2.5 times greater and 2.5 times smaller than ours.

The chief difference in LBW and PRH is in the treatment of the AML (angular momentum loss) SXTs. In the spiral in these overfill their Roche Lobes with $\lambda\alpha_{ce} = 0.2$, presumably even more so with $\lambda\alpha_{ce} = 0.08$. In the donor mass considered ($M_d > 0.7M_\odot$) of LBW the donor did not overfill its Roche Lobe by much. [7] It was assumed that the system adjusted itself quickly by transfer of a small amount of mass to the He star, which widened the orbit until the donor filled its Roche Lobe exactly. In fact we now believe it more likely that the overfill mass is expelled in the common envelope evolution. Meyer & Meyer-Hofmeister (1979) suggest that the common envelope is not expelled until the separation of the inner cores (He-star and donor) has become so small that the dense layers of the donor are finally affected by the tidal interaction. "At this point in the evolution a large amount of mass is rather suddenly released from the main sequence star into the common envelope and the neighborhood of the degenerate binary companion."

PRH check whether the secondary star overfills its Roche Lobe. If so they "assume that the secondary merges with the core and do not follow the binary further." This is presumably the reason that they are unable to evolve the AMLs with their smaller $\lambda = 0.08$, in which case their a_f following common envelope evolution would be 2.5 times smaller than ours. We believe that the great regularity in the 8 AMLs supports our $\lambda\alpha_{ce} \sim 0.2$.

One cannot pin down λ separately from this combination, however, Dewi & Tauris (2001) suggest a lower limit of $\lambda = 0.2$, whereas for deeply convective giants and no wind loss $\lambda \sim 1$ as discussed in our Appendix. We can only guess that $\lambda \sim 0.5$, in the middle of the allowed interval. This would give $\alpha_{ce} \sim 0.4$, saying that the material released in the common envelope evolution has a kinetic energy 2.5 times the averate kinetic energy it had in the initial giant. (Even though some of the released matter comes from the donor through tidal interaction, most of it must come from the envelope of the giant.) Most of this kinetic energy comes when the tidal interaction has cut strongly into the donor.

In the best calculation of common envelope evolution to date, Rasio & Livio

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[7] And with slightly larger $\lambda\alpha_{ce}$ it would not overfill the Roche Lobe at all.

(1996) say that perhaps their most significant new result is that during the dynamical phase of CE evolution, a corotating region of gas is established near the central binary. This is done through a combination of spiral shock waves and gravitational torques that can transfer angular momentum from the binary orbit to the gas. The corotating region has the shape of an oblate spheroid encasing the binary (i.e., the corotating gas is encased in the orbital phase). The assumption that rigid rotation is tidally enforced in a core surrounding the inner binary was already made by Meyer & Meyer-Hofmeister (1979). Rasio & Livio (1996) did not carry their calculation beyond this stage.

LBW assumed that at least the outer part of the He star is isochronous with the donor at the time of the explosion forming the black hole. In fact, the giant will probably have been brought into common (rigid body) rotation by the dynamo process proposed by Spruit (2001) and even more so by the magnetic field modeling suggested by Spruit & Phinney (1998). In general this rotation will be about an axis different from that established later in the common envelope evolution. The assumption that rigid rotation is tidally enforced in a core surrounding the inner binary was already made by Meyer & Meyer-Hofmeister (1979). However, the common envelope time is short, the dynamical time of years, so it would be expected to bring only the outer part of the He core into synchronism. The remaining He-star burning following common envelope evolution of ~ 100 years seems too short to effect synchronization by itself. However, in this time the inner core can pull away from the outer He star (Spruit & Phinney). Thus, we believe that sufficient differential rotation will be achieved to allow the center of the He star to fall into a black hole and the surrounding part into an accretion disk.

Wilson (1989) has examined synchronism in Algol systems. Out of 33 systems, about 2/3 show synchronism for periods less than two weeks.

With a λ-parameter of 0.5 PRH find a formation rate for binaries with $m_d < 2M_\odot$, the limit for donors which remain in main sequence, of 7×10^{-7} yr^{-1}. They normalize to a supernova rate of 1 per century. We use a rate of 3 per century, giving a formation rate of 2.1×10^{-6} yr^{-1}. Although these live a Hubble time, we can observe these for 10^9 yrs (Lee & Brown 2002) so this would give 2100 presently observable in the Galaxy, of the same general number as Wijers' estimate of 3000 (Wijers 1996).

PRH give $\gtrsim 10^{-5}$ as the formation rate for binaries with $m_d < 15M_\odot$. We wish to take an upper limit of at least $20M_\odot$, to include Cyg X-1, and to multiply their formation rate by our 3 normalization factor. We find them a formation rate of $\sim 5 \times 10^{-5}$ yr^{-1}, a factor ~ 4 less than the estimate of Lee & Brown (2002), but as noted there, a realistic number for those which could be relics of GRBs might be as low as $\sim 10^{-5}$ galaxy^{-1} yr^{-1}. In other words, the progenitor binaries of the black-hole binaries are sufficient to also

be progenitors of GRBs.

6   Discussion

The work of LBW (Lee, Brown, & Wijers 2002) has been amalgamated with that of PRH (Podsiadlowski et al. 2002). Both papers agree that common envelope evolution must come following helium core burning; i.e., be Case C. Our work is based on the nearly linear relationship Eq. (1) between separation following common envelope evolution and companion (donor) mass, which is spelled out in our Appendix. By choosing the common envelope parameter $\lambda\alpha_{ce} = 0.2$, we are able to evolve those binaries with K and M-star companions, which we believe to be the success of LBW and our earlier works.

The PRH evolutions clarify that all of the binaries with the possible exception of Cyg X-1 could have made Roche contact in main sequence evolution of the companion, as in the earlier work by Beer & Podsiadlowski (2002). This makes it possible to evolve all of the binaries with sub-Eddington rate of mass transfer (although we do not believe this to be necessary in the case of black holes).

LBW and PRH agree that the present evolutionary tracks of supergiants from ZAMS masses $\sim 20 - 30 M_\odot$ and possibly greater, must be changed so as to allow Case C mass transfer. An example of how to do this was constructed (by hand) in LBW.

We would like to point out that there are uncertainties due to the radius evolution of massive stars and the parameterization of mixing, etc. In particular, stars that use the LeDoux criterion or a small amount of semiconvection burn helium as red supergiants, while those with Schwarzschild or a lot of semiconvection stay blue much of the time. Rotationally induced mixing also plays a role as in Langer and Maeder (1995).

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A Common Envelope Efficiency

The binding energy of the envelope is parameterized by λ (Webbink 1984),

$$E_{\mathrm{env}} = -\frac{GM_{\mathrm{giant}}M_{\mathrm{env}}}{\lambda R}, \tag{A.1}$$

where M_{env} is the mass of the envelope in the giant star. During the common envelope evolution, this binding can be compensated by the total change in binary orbital energy with efficiency α_{ce}

$$E_{\mathrm{env}} = \alpha_{ce}\left(-\frac{GM_{\mathrm{He}}M_{\mathrm{d}}}{2a_f} + \frac{GM_{\mathrm{giant}}M_{\mathrm{d}}}{2a_i}\right), \tag{A.2}$$

where $a_{i,f}$ are the initial and final orbital separations and M_{comp} is the donor companion mass, and one can neglect last term with a_i. Hence, a_f can be expressed as

$$
\begin{aligned}
a_f &\approx \lambda\alpha_{ce}M_{\mathrm{d}}R\frac{M_{\mathrm{He}}}{2MM_{\mathrm{env}}}\\
&\approx 0.04\ \lambda\alpha_{ce}\frac{M_d}{M_\odot}\left(\frac{M_{\mathrm{giant}}}{M_\odot}\right)^{-0.55}R \times \frac{M_{\mathrm{giant}}}{M_{\mathrm{env}}}
\end{aligned}
\tag{A.3}
$$

where we have used $M_{\mathrm{He}} = 0.08(M/M_\odot)^{1.45}M_\odot$. The final factor $M_{\mathrm{giant}}/M_{\mathrm{env}} \simeq 3/2$ to within 5% over our range of ZAMS masses $20 - 30M_\odot$. Note that only the combination $\lambda\alpha_{ce}$ is relevant. In LBW in the commonly used Webbink formalism we found, analyzing the same orbits, $\lambda\alpha_{ce} = 0.2$. Here λ is a parameter taking into account the polytropic structure of the giant. For deeply convective giants without mass loss, Brown et al. (2001b) found $\lambda \simeq 7/6$ in their Appendix C.

In order to get the estimated values of λ of the mass losing star (Dewi & Tauris 2001), we calculated the binding energy parameter λ for the convective (polytropic index $n = 3/2$) and radiative ($n = 3$) envelopes in Fig. A.1. In this figure, we considered various values of fractional core mass. Note that the simple formula used in our calculation $M_{\mathrm{He}} = 0.08M^{1.45}$ gives initial values of x to be $0.3 - 0.4$ for the ZAMS mass range $20 - 40M_\odot$. However, the fractional core mass will increase during the common envelope evolution. In the estimation of λ of the mass losing star in Fig. A.1, we assumed that i) the size of core is negligible compared to the radius of the star R, ii) the structure of the original star doesn't change when we remove part of envelope outside of radius r. So, the plotted λ is the binding energy parameter of the remaining star inside radius r.
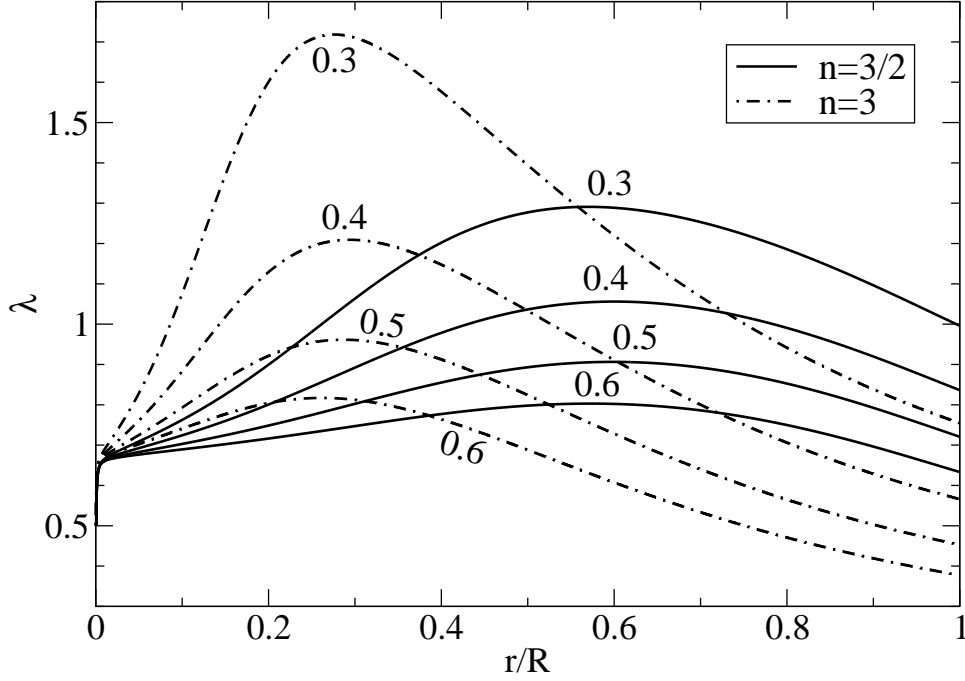
Fig. A.1. Estimated λ parameter of the mass losing star with convective (n=3/2) and radiative (n=3) envelopes. The numbers in the plot are the mass ratios of the core to the total star mass, x. Note that the He core mass fraction of ZAMS $20 - 40 M_\odot$ star is $0.3 < x < 0.4$ with assumed formula $M_{\mathrm{He}} = 0.08 M^{0.45}$. However, x will increasing while losing mass, and larger x is more relevant for the later stages of the common envelope evolution. Since the star changes structure while losing mass and the size of core is not negligible for small r, the physically relevant region will be near $r/R \sim 1$.

Since the star changes structure while losing mass and the core size is not negligible for small radius, the physically relevant region will be large radius near $r/R \sim 1$. For the fully convective star with $x = 0.4$, the binding energy parameter $\lambda \sim 0.75$. Since the fractional mass ratio x increases during the common envelope evolution and the outer radiative ($n = 3$) part of the envelope will decrease λ, lower average $\lambda \sim 0.5-0.7$ is a reasonable approximation for the common envelope evolution. So the efficiency parameter becomes $\alpha_{ce} \sim 0.3 - 0.4$ to have $\lambda \alpha_{ce} = 0.2$ found in LBW.

References

Amati, L., et al. 2000, Science 290, 953

Beer, M. and Podsiadlowski, Ph. 2002, *Mon. Not. of Royal. Astron. Soc.* **331**, 351.

Brown, G.E., Heger, A., Langer, N., Lee, C.-H., Wellstein, S., and Bethe, H.A. 2001a, *New Astronomy* **6**, 457.

Brown, G.E., Lee, C.-H., Portegies Zwart, S.F., and Bethe, H.A. 2001b, *Astrophysical Journal* **547**, 345.

Brown, G.E., Lee, C.-H., and Tauris, T. 2001c, *New Astronomy* **6**, 331.

De Kool, M., van den Heuvel, E.P.J., and Pylyser, E. 1987, *Astron. and Astrophys.* **183**, 47.

Dewi, J. D. M. and Tauris, T. M. 2001, Proc. of "Evolution of Binary and Multiple Star Systems", ASP Conference Series, Vol. 229, eds. Ph. Podsiadlowski, S. Rappaport, A. R. King, F. D'Antona, and L. Burderi, p. 255.

Fryer, C.L., Heger, A., Langer, N., and Wellstein, S. 2002, *Astrophysical Journal* **578**, 335.

Garcia, M.R., McClintock, J.E., Narayan, R., and Callanan, R.J. 1997, Proceedings of the 13th North American Workshop on CVs, eds. S. Howell, E. Kuulkers, C. Woodward (San Francisco: ASP) 506 (1997); astro-ph/9708149.

Langer, N. and Maeder, A. 1995, *Astron. and Astrophys.* , **295**, 685.

Lee, C.-H. and Brown, G.E. 2003, Int. J. of Mod. Phys. **A 18**, 527.

Lee, C.-H., Brown, G.E., and Wijers, R.A.M.J. 2002, *Astrophysical Journal* **575**, 996 (LBW).

Meyer, F. and Meyer-Hofmeister, E. 1979, *Astron. and Astrophys.* **78**, 167.

Nelemans, G. and van den Heuvel, E.P.J. 2001, A&A, **376**, 950.

Podsiadlowski, Ph., Rappaport, S., and Han, Z. 2003, MNRAS **341**, 385 (PRH).

Portegies Zwart, S.F., Verbunt, F., and Ergma, E. 1997, *Astron. and Astrophys.* **321** 207.

Rasio, F.A. and Livio, M. 1996, *Astrophysical Journal* **471**, 366.

Schaller, G., Schaerer, D., Meynet, G., and Maeder, A. 1992, *Astron. and Astrophys. Suppl.* **96**, 269.

Tauris, T.M. and Savonije, G.J. 1999, A&A, **350**, 928.

Webbink, R.F. 1984, *Astrophysical Journal* **277**, 355.

Wijers, R.A.M.J. 1996, Evolutionary Processes in Binary Stars, Eds. R.A.M.J. Wijers et al. Kluwer, Dordrecht, pp. 327-344.

Woosley, S.E., Langer, N., and Weaver, T.A. 1995, *Astrophysical Journal* **448** 315.

Woosley, S. E. & Weaver, T. A. 1995, ApJS **101**, 181